***A R DIGITECH***
*International Journal Of Engineering, Education And Technology (ARDIJEET)*
*www.ardigitech.in ISSN 2320-883X, VOLUME 10 ISSUE 04 01/10/2022*

# BLACK HOLES SHADOW: ANALYTICAL DESCRIPTION IN PHYSICS

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**ABSTRACT**

Einstein presented the special theory of relativity where he determined that the laws of physics appear same in all inertial frames and the speed of light is constant in vacuum. After the discovery of special theory of relativity, Einstein took another ten years to include acceleration in his theory and in 1915 he established the general theory of relativity for an accelerating observer. The general relativity (GR) is a theory of gravity in which Einstein presented a unified picture of space and time, well known as spacetime which was abandoned by Newtonian gravity. One of the most interesting and intriguing prediction of GR is the existence of black holes in our universe. Black holes are the region in the spacetime where the gravitational pull is so high that even light cannot escape from it. Black holes are the most exotic astrophysical objects due to its unique behavior and a black hole solution can be found by solving Einstein field equations.

**KEYWORD**: Mathematical physics, Physical Sciences, Physics, Physics Mathematical, Space and time

## INTRODUCTION

Black holes have very rich physics not only in astrophysical but also in theoretical point of view. The idea of the escape velocity from the surface of a compact object greater than the velocity of light was first discussed by John Mitchell. After that in 1796, Pierre Simon de Laplace investigate this idea. In the 20th century, just after the discovery of GR, a German mathematician Schwarzschild exactly solved the Einstein's field equation for vacuum to find first black hole metric namely Schwarzschild metric. The Schwarzschild metric contains two types of singularity, a central singularity at its center $r = 0$ and a removable coordinate singularity at its Schwarzschild radius $rs = 2GM$. It was later discovered the limit of maximum mass of a white dwarf often called the Chandrasekhar's limit. When a white dwarf star crosses. it will become a neutron star or a black hole by further gravitational collapse. Fritz Zwicky and Walter Baade studied all the properties of white dwarf except its maximal mass and after that a lot of people worked on the maximal mass of the white dwarf including Landau. Oppenheimer and Volkoff showed the upper bound limit of a massive star in the formation of neutron star and again Oppenheimer collaborated with Synder and studied the gravitational collapse for a massive star which forms black holes. The formation of black holes from a collapsing sufficiently massive star takes infinite time for a far distant observer, so at that time collapsing star called frozen star. Almost forty years after Schwarzschild black hole solution, Kerr derived the rotating counterpart of Schwarzschild metric. Wheeler gave the physical model of collapsing star and first introduced the term "black holes" in his paper.

The matter composition of black holes is highly compressed that's why gravity is so strong around it. Any external matter which comes in the gravitational field of the black hole it crosses the event horizon and fall inside the black hole or accumulate around the black hole in the form of an accretion disc. The event horizon is the boundary of the black hole and any kind of matter lost for forever.

When the matter of an object contract to Schwarzschild or gravitational radius which is rs = 2GM/c2 (where G is the gravitational constant, M is the mass of the object and c is the velocity of the light), it becomes a black hole. The escape velocity of a black hole is the velocity of an object to leave the event horizon which is equal to the speed of light. As we know that nothing can go faster than light so it is impossible for an object to leave the boundary of a black hole. Any black hole can be characterized by three name parameters, mass (M), spin (a) and charge (q) go by a name No-hair theorem. The mass and the spin of a black hole are the main parameters to define its physical properties. Nowadays there are so many experiments have been done to verify black holes in our Universe, such as detection of the gravitational waves from merging of two black holes by LIGO (laser interferometry gravitational observatory) and gravitational lensing around the compact sources. In what follows, we discuss the past and upcoming observations in the context of black holes.

**Observations for black holes**

Much theoretical work has been done in the field of black holes till the end of the sixties and after that astrophysicists and astronomers showed his interest in this subject and start taking observations on black holes. Novikov and Shklosky studied the accretion of matter around black hole from electromagnetic radiation. In the study of accretion of matter Shklosky and Brecher have found that the black holes are a good emitter of X-rays. The work of Blandford and Rees suggested that the active galactic nuclei and quasars are the supermassive black hole candidate. Fabian et. al. studied shapes of electromagnetic spectral lineemitted from the black hole. At the end of the 20th century, so many astronomers did a lot of observations and confirmed that a supermassive black hole exists at the core of every galaxy in the universe. On the basis of astronomical observations, the black hole mass divided into a wide range of spectrum and can be classified as

• Supermassive black holes (M ∼ 105 − 109MJ)

• Intermediate black holes (M ∼ 103MJ)

• Stellar black holes (M ∼ 3MJ)

• Primordial black holes (M ∼ MJ)

**Black hole solutions**

The black hole uniqueness theorem and No-hair theorem direct the properties of fourdimensional black hole solutions of GR in either a vacuum or coupled with an electromagnetic field. In particular, such black holes are either non-rotating and spherically symmetric or rotating and axisymmetric. On the basis of these characterization, we classify black holes in the following way:

• Non-Rotating

1. Schwarzschild black holes (M)

2. Reissner-Nordstrom black holes (M, q)

• Rotating black holes

1. Kerr black holes (M, a)

2. Kerr-Newman black holes (M, a, q)

**Non-rotating black holes**

The non-rotating black holes are characterized by its mass and its charge. Astrophysical black holes are rotating but when their spin turns off they considered as static or non-spinning black holes. The simplest form of static black hole is the Schwarzschild black hole which we discuss next.

**Geodesics in Schwarzschild spacetime: timelike geodesics**

The black hole metric holds two killing vectors, $\xi_t^\mu$ and $\xi_\phi^\mu$. These two Killing vectors corresponds to two conserved quantities energy E and magnitude of the angular momentum L. For Schwarzschild black hole metric, these conserved quantities can be defined as

$$E = -g_{\mu\nu}\xi_t^\mu u^\nu = -g_{tt}u^t = \left(1 - \frac{2M}{r}\right)\frac{dt}{d\sigma}, \qquad (1.5)$$

$$L = g_{\mu\nu}\xi_\phi^\mu u^\nu = g_{\phi\phi}u^\phi = r^2 \sin^2\theta \frac{d\phi}{d\sigma}, \qquad (1.6)$$

where $u^\nu$ is the four-velocity of the particle and $\sigma$ is the affine parameter. The most general form of timelike geodesics equation can be expressed as

$$-g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma} = 1, \qquad (1.7)$$

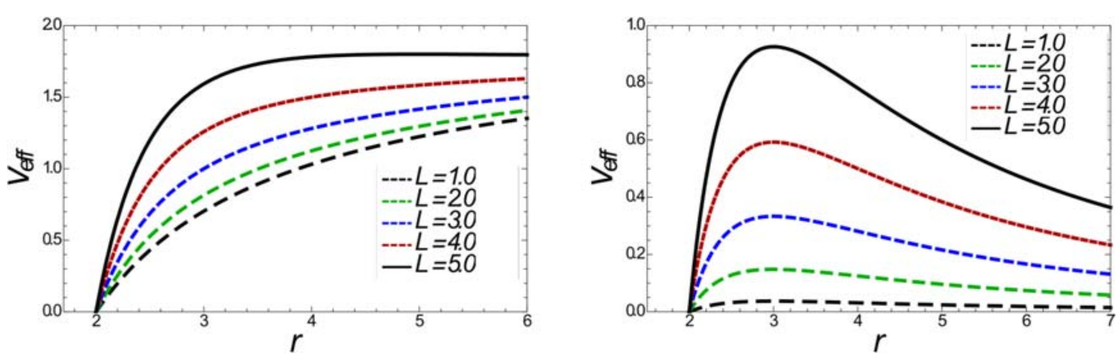


Figure 1.1: Plot showing the behavior of effective potential $V_{eff}$ with $r$ for massive (left) and massless particle (right).

on equatorial plane ($\theta = \pi/2$), one can expand the timelike geodesics equation (1.7) in the following form

$$\left(1-\frac{2M}{r}\right)\left(\frac{dt}{d\sigma}\right)^2-\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{dr}{d\sigma}\right)^2-r^2\left(\frac{d\phi}{d\sigma}\right)^2=1. \tag{1.8}$$

By using equations (1.5) and (1.6), we can get the timelike radial geodesics in terms of energy E and angular momentum L, which reads

$$\left(\frac{dr}{d\sigma}\right)^2+\left(1-\frac{2M}{r}\right)\left(1+\frac{\mathcal{L}^2}{r^2}\right)=\mathcal{E}^2. \tag{1.9}$$

The above equations (1.5), (1.6) and (1.9) are the timelike geodesics for the Schwarzschild black hole.

## Rotating black holes

The astrophysical observations conforms that most of the stars have some angular momentum so the rotating black holes are the astrophysical black holes candidate. Ordinarily, an ancestor massive star has a non-vanishing angular momentum. Even if an essential part of a massive star is lost during the process of the black hole formation, the afresh born black hole would be rotating. But even if a black hole is basically formed without angular momentum or with a small value of it, it will acquire angular momentum as an outcome of the interaction with the matter around it. The rotating black holes are completely defined by the Kerr metric and next we briefly discussed the Kerr black hole metric and its properties.

### Kerr metric

The Kerr metric is an exact solution of the Einstein field equations for a stationary and axially symmetric spacetime in vacuum [15]. The Kerr black hole metric in Boyer-Lindquist coordinates, reads [15, 40]

$$\begin{aligned} ds^2 = &-\left(1-\frac{2Mr}{\Sigma}\right)dt^2-\frac{4Mar\sin^2\theta}{\Sigma}dtd\phi+\Sigma d\theta^2+\frac{\Sigma}{\Delta}dr^2 \\ &+\sin^2\theta\left(r^2+a^2+\frac{2Ma^2r\sin^2\theta}{\Sigma}\right)d\phi^2, \end{aligned} \tag{1.20}$$

where

$$\Delta = r^2 - 2Mr + a^2, \qquad \text{and} \qquad \Sigma = r^2 + a^2\cos^2\theta.$$

Here, the parameter $M$ and $a$ are the black hole mass and spin parameter ($a = J/M$) respectively. The spin parameter $a$ has the units of length in terms of geometric units ($c = G = 1$). The necessary condition for the existence of the event horizon in case of rotating

Kerr black hole is $|a| \leq M$ otherwise the spacetime will form a naked singularity. The Kerr black hole metric (1.20) reduces to the Schwarzschild metric (1.2) in the absence of spin parameter ($a = 0$).

The Kerr black hole metric possess two Killing vectors $\xi_t^\mu = (1,0,0,0)$ and $\xi_\varphi^\mu = (0,0,0,1)$ due to the assumptions of stationary and axisymmetric spacetime. The metric tensor $g_{\mu\nu}$ of metric (1.20) is independent of time coordinate $t$ and azimuthal coordinate $\varphi$ and remains the functions of other two coordinates so that

$$g_{\mu\nu} = g_{\mu\nu}(r,\theta). \tag{1.21}$$

The black hole metric remains invariant in the joint inversion of the $t$ and $\varphi$ coordinates i.e. under the following transformations

$$(t,\varphi) \rightarrow (-t,-\varphi) \tag{1.22}$$

the black hole metric remains unchanged.

#### 1.3.2.2 Horizons and singularities

It can be seen from the metric (1.20) that the line element goes to infinity $ds^2 \rightarrow \infty$ at $\Delta = 0$, which is a coordinate singularity and corresponds to the horizons of the black hole. The horizons of the Kerr black hole satisfies the following condition

**Ergosphere and static limit surface**

The ergosphere or egoregion of a rotating black hole is the region between the event horizon and the static limit surface (cf. Fig. 1.2). This region rotates very fast along with black hole and due to this it is impossible for a particle to remain stationary in this region. The four-velocity for a stationary observer can be defined as

$$u^\mu = (u^0, 0, 0, 0) = \left(\frac{dt}{d\tau}, 0, 0, 0\right). \tag{1.26}$$

One can take the magnitude of four velocities for the black hole metric (1.20) in the given form

$$u.u = g_{00}(u^0)^2 = -1, \tag{1.27}$$

where

$$g_{00} = -\left(1 - \frac{2Mr}{\Sigma}\right) = -\left(\frac{r^2 + a^2\cos^2\theta - 2Mr}{r^2 + a^2\cos^2\theta}\right). \quad (1.28)$$

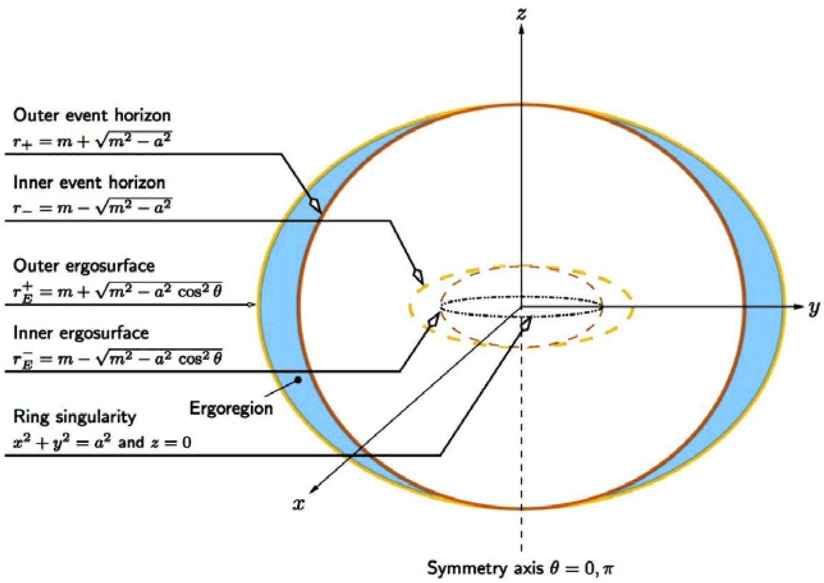


Figure 1.2: Schematic representation of Kerr black holes [44].

The above $g_{00}$ component of the metric vanishes if

$$r^2 - 2Mr + a^2\cos^2\theta = 0, \quad (1.29)$$

by solving we obtain

$$r_e(\theta) = M \pm \sqrt{M^2 - a^2\cos^2\theta}. \quad (1.30)$$

The region between the $r_e(\theta)$ and the event horizon $r_+$ defines the ergoregion of the rotating black hole. When $a = 0$, the two horizons of the Kerr black holes reduces to the single event horizon of the Schwarzschild black hole and in that case the $r_e(\theta)$ surface overlap with the $r_+$ surface.

## CONCULSION

GR fits not only on the large scale but it also works in the weak field limit. GR already passed almost all the experimental tests in weak field limit which verifies that the GR is the standard theory of gravity but this theory is still untested in the strong field region. As we know that the gravitational field near the black hole is so strong and if we want to test GR in the strong field region then its better to test it around a supermassive black hole. The EHT network is currently trying to get the image of black hole shadow and the detection of the black hole image will verify the GR in the strong field regime. The analytical study of black hole shadow in the alternative theory of gravity is also important due to the upcoming observation from EHT. From the observed shadow image by EHT, we can compare our theoretical model and test GR against alternative or modified theories of gravity. The size of the black hole shadow decreases and the distortion increases with the increasing values of charge q with normalization factor c. For the better understanding of black hole shadow, we also studied its shadow observables Rs and δs. The study of energy emission rate of the black hole also has been done. The study of black hole shadow is the most effective tool to observe the spin parameter and the very first image of black hole shadow will not only verify the event horizon of the black hole but even also it will test GR in the strong field region.

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